Economics 521 Spring, 2005 First exam.

Do all five problems. EXPLAIN your reasoning for each answer!! [Tell me WHY you add when you add, WHY you multiply when you multiply, ...] Include a statement of exactly what assumptions you are making.

1. (20 pts) Consider a fair die that we are going to toss 20 independent times.
   (A) Which of the following sequences is more likely
   5 4 3 6 1 4 2 6 3 1 5 4 6 2 3 5 4 3 6
   or
   5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
   (B) Which is more likely, getting three 5s in 20 tosses or getting four 5s?

2. (20 pts) You toss a fair coin until the first occurrence of “heads”; then I independently toss a fair coin until it comes up heads. What is the probability that the two sequences have the same length? [Hint: what is the probability that we both get heads on the first toss? What is the probability that we both get the sequence TH? Etc.]

3. (20 pts) For the experiment of dealing a five card hand from a well-shuffled poker deck, the random variable X is the number of denominations represented in the hand. (Recall there are 13 denominations altogether: 2, 3,..., Q, K, A.) Determine the probability distribution of X (15 pts) and calculate the mean of this random variable (5 pts).

4. (20 pts) Professor Lovely has a class with three students: Aaron, Benita, and Charles. All three forgot to put their names on last week's homework, which consisted of three problems. Generally, Aaron solves any one homework problem correctly with probability .9; the corresponding probabilities for Benita and Charles are .7 and .5. Professor Black picks one of the homework papers at random and grades it.
   A. What is the probability that two of the three problems are incorrect if the homework was done by Aaron? By Benita? By Charles?
   B. What is the probability that the homework was done by Aaron if two of the three problems are incorrect?
   C. What is the probability that the homework was done by Benita if two of the three problems are incorrect?
   D. What is the probability that the homework was done by Charles if two of the three problems are incorrect?

5. (20 pts) For the experiment of tossing a fair die n times, the random variable X is the maximum (largest) number that turns up. For example, with the sequence
   2 3 4 3 1 5 2 4 3 1 2 4 5 1 1 2 4 6 2 2
   X = 5 (since a 5 appears and no 6 does). Determine the probability distribution of X (your answer will depend on n, the number of tosses).