Economics 611  Handout #6

THE ECONOMICS OF ASYMMETRIC INFORMATION

Chap. 13: Asymmetric info at the time of contracting: Adverse Selection and Screening

Model I. [First just an equilibrium model \textbf{without} any game-theory flavor]:

Competitive Output market; \( p = 1; \ w = p \cdot \text{MPP} = 1 \cdot \theta \)

Two kinds of workers: low and high productivity: \( \theta_L < \theta_H \); Numbers: \( N_L, N_H; \ N_L + N_H = N \)

\[
\bar{\theta} = \frac{N_L \theta_L + N_H \theta_H}{N_L + N_H}
\]

Opportunity cost of employment: \( r_L, r_H \)

Two cases:

I. \( \theta_i \)'s are observable: At equilibrium, \( w_L = \theta_L; w_H = \theta_H \).

II. Workers know own \( \theta_i \); \( \theta_i \)'s are unobservable by firms. [So single wage, \( w \).]

Supply: \( N \) of \( \theta_i \) offer to work if \( w \geq r \) [tie issue] \( \Theta(w) = \{ \theta : w \geq r(\theta) \} \)

Demand: Let \( \mu = \) expected average productivity of those who accept employment. [This is ASSUMED to be \( \bar{\theta} \) if \( \Theta(w) \) is empty.]

\[
\mathcal{Z}(w) = \begin{cases} 
0 & \text{if } \mu < w \\
[\theta, \infty) & \text{if } \mu = w \\
\infty & \text{if } \mu > w 
\end{cases}
\]

Definition (Akerlof): \((w^*, \Theta^*)\) is a competitive equilibrium if

(i) \( \Theta^* = \Theta(w^*) \);

(ii) \( w^* = E(\theta : \theta \in \Theta^*) \)
A. It is possible that such an equilibrium is NOT Pareto optimal.

Example: \( \theta_L < r_L = r_H < \theta_H \)  \[ r = r_L = r_H \]

Then at any Akerlof equilibrium,

\[
\Theta^*(w) = \begin{cases} 
\{\theta_L, \theta_H\} & \text{if } w \geq r; \\
\emptyset & \text{if } w < r.
\end{cases}
\]

Either way, \( E(\theta : \theta \in \Theta^*) = \bar{\theta} \); so at equilibrium, \( \bar{\theta} = w^* \).

There must be unrealized contracts:

Case 1. \( \bar{\theta} > r \). Then \( w^* > r \) and everyone works. Note: \( w^* - r < w^* - \theta_L \). A firm would like to offer a \( w'' \) between these: \( w^* - r < w'' < w^* - \theta_L \) to a Type-L to quit since then \( w^* - w'' > \theta_L \).

The Type-L would like to take this offer since \( r + w'' > w^* \).

Case 2. \( \bar{\theta} < r \). Then \( w^* < r \) and no one works. A firm would like to offer a \( w' \), \( r < w' < \theta_H \), to a Type-H to work and a Type-H would like to take this offer.

B. It is possible there is no equilibrium: Adverse selection and market unraveling:

\( r(\theta) \) varies with \( \theta \). Here: \( r_H > r_L \).

\( N_L = 100 = N_H; \theta_L = 1/3; r_L = 3/7; r_H = 4/7; \theta_H = 2/3. \) Note: \( \theta_L < r_L < r_H < \theta_H \)

(1) Suppose \( \Theta^* = \{\theta_L, \theta_H\} \)  \( E(\theta : \theta \in \Theta^*) = [100 \cdot (1/3) + 100 \cdot (2/3)] / [100 + 100] = 1/2 \)

If \( w = 1/2 \), \( w > r_L \) but \( w < r_H \); so \( \Theta(w) = \{\theta_L\} \) and only Type-L workers accept employment.

(2) Suppose \( \Theta^* = \{\theta_L\} \) then \( E(\theta : \theta \in \Theta(w)) = \theta_L = 1/3 < 3/7 \) and \( \Theta(w) = \emptyset \); Type-L workers also refuse employment.

There does not exist an Akerlof equilibrium.

That requires also looking at (3) \( \Theta^* = \{\theta_H\} \) and (4) \( \Theta^* = \emptyset \).
Model II. [now with game theory flavor]
Continuous case: Common knowledge: \( \theta \) distributed uniformly on \([1, 2]\); \( r(\theta) = .9\theta \)

So, for example, 
\[
E(\varnothing / r(\theta) \leq 1) = E(\varnothing / 1 \leq \theta \leq 1.111...) = 1.05555...
\]

and 
\[
E(\varnothing / r(\theta) \leq 1.8) = E(\varnothing / 1 \leq \theta \leq 2) = 1.5. E(\varnothing / r(\theta) \leq w) = \frac{1}{2} (1 + w/.9) \text{ for } .9 \leq w \leq 1.8
\]

An Akerlof equilibrium is found at \( w^* = 1.125 \) and \( \Theta^* = \{ \varnothing / 1 \leq \theta \leq 1.25 \} \)

We want to show this results from a pure subgame perfect Nash equilibrium.

The SPNE strategies are:

(i) For workers: A worker of type \( \theta \) accepts employment only at one of the highest wage firms and does so if and only if \( r(\theta) \leq w^* \) where \( w^* \) is the highest wage offered.

(ii) Both firms offer wage of 1.125
Clearly if \( r(\theta) > 1.125 \), switching to accepting employment at 1.125 makes the worker worse off. If \( r(\theta) \leq 1.125 \), a worker can not be made better off by switching to not accepting work, or by switching to working at a lower wage firm.

Notice that since \( E( \theta / r(\theta) \leq 1.125) = 1.125 \), the firms are both earning a profit of 0. If a firm switches to offer less than 1.125, they get no workers, and get a profit of 0, no better than before. If a firm switches to offer \( w' > 1.125 \), they get all the workers and profit per worker is \( E( \theta / r(\theta) \leq w') - w' < 0 \), worse than before.

**Q:** For whom are there unrealized contracts?

**Exercises:** 13B2, 13B5