Economics 611   Handout #9

THE ECONOMICS OF ASYMMETRIC INFORMATION

Chap. 14: Asymmetric info after the time of contracting: Moral Hazard

Standard example: Principal-agent model
“A valid contract cannot commit either party to transfer money under conditions that cannot be independently verified by both.”

Employment model

Employees

HOMOGENEOUS

Effort: \( E, \quad 0 \leq E \leq 1 \)

Von Neumann-Morgenstern utility function:

\[
U = W \cdot (1 - E)
\]

Reservation utility: \( W > 0 \) (Exogenous)

Probability caught shirking: \( \pi(E) = 1 - E \)

Firm

Revenue function: \( V(L \cdot E) = V(L \cdot E) \)

\[
V(0) = 0; \quad V' > 0, \quad V'' < 0
\]

Profit function: \( R(W, L) = V(L \cdot E) - W \cdot L \)
We seek a subgame perfect Nash equilibrium via backwards induction:

**Employee:** \( \max_E \mathbb{E}[U(W, E)] = \pi(E) \cdot W + (1 - \pi(E)) \cdot W \cdot (1 - E) \)

\[ = (1 - E) \cdot W + E \cdot W \cdot (1 - E) \]

**Firm:** Choose \( W \) and \( L \) (or \( L_E \)) to maximize \( R \).

\[ R(W, L) = \begin{cases} 0 & \text{if } W \leq W^* \\ V(L \cdot E^* (W)) - W \cdot L & \text{if } W > W^* \end{cases} \]

**Example:** \( V(L_E) = (L_E)^{1/2} \).

\[ W^* = 2W; \quad L^* = [64W^2]^{-1}; \quad E^* = 1/4. \]

At a solution, \( W^* \), we must have \( W^* > W \).

But \( W^* \) is a market clearing wage, i.e., at full employment, \( W = W^* \). Therefore, the market clearing wage is NOT part of a subgame perfect Nash equilibrium.

**Exercise:** Do it all again with improved detection: \( \pi(E) = 1 - E^2 \).
Unemployment Insurance - - without Moral Hazard

Assume a risk-averse individual with income Y faces unemployment with fixed probability $\pi$.
Insurance option: Indemnity, I ; Premium, P.

*Individual* maximizes expected utility: $U(\text{Net Income})$, $U' > 0$; $U'' < 0$

$$E(U(\text{Net Income})) = \pi(U(I - P) + (1 - \pi) U(Y - P))$$

*Insurance Co.*: Expected profit $R(P,I) = \pi(P - I) + (1 - \pi) P$

Actuarially fair: Expected profit $= 0$: $P^*(I) = \pi I$

*Individual*:

$$E(U(\text{Net Income})) = \pi (U(I - \pi I) + (1 - \pi) U(Y - \pi I))$$

Derivative with respect to I:

$$\pi U'(I - \pi I) (1 - \pi) + (1 - \pi) U'(Y - \pi I) (1 - \pi)$$

Setting equal to 0 at solution $I^*$:

$$\pi (1 - \pi) U'(I^* - \pi I^*) = (\pi) (1 - \pi) U'(Y - \pi I^*)$$

so:

$$U'(I^* - \pi I^*) = U'(Y - \pi I^*)$$

$\therefore$ by monotonicity of $U'$, $I^* = Y$, full insurance.

Example, $\pi = .25$, $Y = 400$ and $U(\cdot) = (\cdot)^{\frac{1}{3}}$: $P = .25 I$ for actuarially fair insurance, so

$$E(U(I)) = .25(I - .25 I)^{\frac{1}{3}} + .75(400 - .25 I)^{\frac{1}{3}}$$
Insurance - with Moral Hazard

Assume a risk-averse individual with income $Y$ faces unemployment with fixed probability (.25) but then can search for a new job, failing in that search with probability $\Phi(E)$ where $E$ is the effort (measured in foregone utility) to prevent that failure.

Insurance option: Indemnity, $I$; Premium, $P$.

**Individual:** If laid off, the expected utility of search is

$$E(U(P,I,E)) = \Phi(E)[U(I - P) - E] + (1 - \Phi(E))[U(Y - P) - E]$$

$$= U(Y - P) - \Phi(E)[U(Y - P) - U(I - P)] - E$$

$$= U(Y - P) - \Phi(E)N(P,I) - E$$

where $N(P,I)$ is the net benefit of finding a new job when holding a $(P,I)$ policy

Solve the maximization problem for $E^*(P,I)$

**Insurance Co.:** Expected profit: $R(P,I) = .25\Phi(E^*(P,I))(P - I) + (1 - .25\Phi(E^*(P,I))) \cdot P$

Set equal to 0 and solve for actuarially fair premium: $P^*(I) = .25\Phi(E^*(P^*(I), I)) \cdot I$

**Individual:** Maximizes

$$E(U(P,I)) = .75U(Y - P) + .25\{(1 - \Phi(E^*(P,I)))[U(Y - P) - E^*(P,I)]$$

$$+ \Phi(E^*(P,I))[U(I - P) - E^*(P,I)]\}$$

$$= U(Y - P) - .25[\Phi(E^*(P,I))N(P,I) + E^*(P,I)]$$

Example: $Y = 400$, $U(\cdot) = (\cdot)^{1/2}$, and $\Phi(E) = e^{-E/4}$ (exponential)

$$E^*(P,I) = \begin{cases} 4 \cdot \ln(N(P,I)/4) & \text{if } N(P,I) > 4 \\ 0 & \text{if } N(P,I) \leq 4 \end{cases}$$

$$P^*(I) = \begin{cases} I N(P^*(I),I) & \text{if } I < 270 \\ 0.25 \cdot I & \text{if } I \geq 270 \end{cases}$$
[270 is the solution to $N(0.25I, I) = 4]$