All Syracuse University policies and procedures concerning academic honesty apply to this course:

"Syracuse University students shall exhibit honesty in all academic endeavors. Cheating in any form is not tolerated, nor is assisting another person to cheat. The submission of any work by a student is taken as a guarantee that the thoughts and expressions in it are the student’s own except when properly credited to another. Violations of this principle include: giving or receiving aid in an exam or where otherwise prohibited, fraud, plagiarism, the falsification or forgery of any record, or any other deceptive act in connection with academic work. **Plagiarism is the representation of another's words, ideas, programs, formulae, opinions, or other products of work as one's own either overtly or by failing to attribute them to their true source.**" (Section 1.0, University Rules and Regulations)

**WARNING!!!**

While homework problems may have been done cooperatively, **exams are individual work.** Do not communicate about this exam with **anyone** except the instructor [x3-2345 or e-mail jskelly@maxwell.syr.edu]. **Violation of this rule will result in a grade of 0 for the exam.** Any notices will be sent to you by e-mail; check occasionally.

**EXPLAIN** your answers carefully.

Take-home part DUE: 9:30 am, Thursday, February 13th, in class.
EXPLAIN your answers carefully. The four problems (including the take-home) are each worth 25 points.

1. There are 5 players, trying to get a part of the $1,000,000 prize. Each names a share: s1, s2, s3, s4, and s5; \( 0 \leq s_i \leq 1 \).

   If \( s_1 + s_2 + s_3 + s_4 + s_5 \leq 1 \), then the players receive the shares they named.
   If \( s_1 + s_2 + s_3 + s_4 + s_5 > 1 \), then all players receive zero.

(Again, utilities are linear in money.)

   A. Describe all the Nash equilibria \((s_1^*, s_2^*, s_3^*, s_4^*, s_5^*)\) in which everyone chooses the same strategy \((s_1^* = s_2^* = s_3^* = s_4^* = s_5^*)\).

   B. Are there any Nash equilibria other than those found in Part A, but where all players receive the same payoff?

2. A. Construct a game in normal form for which there exists a Nash equilibrium \((s_1, s_2, s_3, ... )\) at which two players, say #1 and #2, if they BOTH change their strategies will both be strictly better off. (No one person can, by themselves, manipulate to a better outcome, but two people together can.)

   B. Construct a 2-person game in normal form for which there does NOT exist a Nash equilibrium in pure or mixed strategies.

3. For the three-person game on the last page, determine all subgame perfect Nash equilibria. Does there exist a Nash equilibrium that is not subgame perfect?

   To aid description of equilibria, each information set has been given a numerical label from 1 to 6 (5 of the information sets are singletons).

   Choice options at information sets are labeled with upper-case letters.
4. Consider a first-price sealed bid auction of an object with two bidders. Each bidder i’s valuation is an integer v_i, which dollar amount is known to both bidders. Assume v_2 > v_1 and v_1 ≥ 0. The auction rules are that each player submits a bid (a non-negative integer) in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object but pays the auctioneer the amount of that highest bid. If both bidders submit the highest bid, each gets the object (and pays their bid) with probability 1/2 (and with probability 1/2 gets nothing and pays nothing). Bids must be in dollar multiples (assume that valuations are also).

So each strategy space is S_i = {0, 1, 2, 3, ...}. If s_i > s_2, the payoffs are (v_i − s_i, 0). If s_2 > s_i, the payoffs are (0, v_2 − s_2). If s_i = s_2, the payoffs are ((v_1 − s_i)/2, (v_2 − s_2)/2).

How many Nash equilibria are there? What are they?

[ Your answer will depend on v_1 and v_2. ]