Normal Form Games

Prisoner’s dilemma:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(1, 1)</td>
<td>(−1, 10)</td>
</tr>
<tr>
<td>D</td>
<td>(10, −1)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

C: cooperate      D: Defect

The closest-to-2/3 game:

Variation 1. There are five players. For each individual $i$, the strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. If players choose $(s_1, s_2, ..., s_5)$, the mean of the $s_i$ values is $m$. Player $i$ wins if she has the minimum, over all $j$, of $|s_j - (2/3)m|$. Then $100 is divided equally among those who win. Are there any Nash equilibria? If so, what are they?

Variation 2. Same as Variation 1, except that, for each individual $i$, the strategy space is $S_i = [0, 10]$, the closed interval of real numbers.

The wall-color game:

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<th>G</th>
<th>W</th>
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<tbody>
<tr>
<td>G</td>
<td>(+1, −1)</td>
<td>(−1, +1)</td>
</tr>
<tr>
<td>W</td>
<td>(−1, +1)</td>
<td>(+1, −1)</td>
</tr>
</tbody>
</table>

Are there any Nash equilibria? If so, what are they?

The NYT bidding game:

Variation 1. There are eight players bidding for a prize of $20. For each individual $i$, the strategy space is $S_i = \{0, .01, .02, ..., 20.00\}$. If players choose $(s_1, s_2, ..., s_8)$, player $i$ wins if $s_i \geq s_j$ for all $j$. If there are $k$ winners, each winner $i$ receives $(20 − s_i)/k$. What are ALL the Nash equilibria of this game?
The greed-punishing game:

**Variation 1.** Players 1 and 2 may split a million dollars. Both players simultaneously name shares they would like to have, $s_1$ and $s_2$, with $0 \leq s_1 \leq 1$ and $0 \leq s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the share they named. If $s_1 + s_2 > 1$, then both players receive zero. (Utilities are linear in money.) What are ALL the Nash equilibria of this game?

**Variation 2.** Players 1 and 2 may split a million dollars. Both players simultaneously name shares they would like to have, $s_1$ and $s_2$, with $0 \leq s_1 \leq 1$ and $0 \leq s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then player #1 receives the fraction $s_1/(s_1+s_2)$ of the million and player #2 receives the fraction $s_2/(s_1+s_2)$ of the million. If $s_1 = s_2 = 0$, both get $500,000$. If $s_1 + s_2 > 1$, then both players receive zero. What are ALL the Nash equilibria of this game?

**Variation 3.** Same as Variation 1, except that there is no upper bound on $s_i$ values, i.e., $s_i$ can be any non-negative real number.

**Simultaneous Auctions**

**Variation 1.** There are five bidders for a rare first edition of Debreu’s *Theory of Value*. Their valuations are $v_1 > v_2 > \ldots > v_5 > 0$. Highest bidder gets the book and pays that highest bid. That is, if player $i$ bids $b_i$, then player $i$ gains $v_i - b_i$ if $b_i$ is the uniquely highest bid, $(v_i - b_i)/2$ if $i$ ties with one other for highest bid, and $(v_i - b_i)/3$ if $i$ ties with two others for highest bid and so on. Is it always a NE for everyone to bid their valuation?

**Variation 2.** Similar, but highest bidder, while getting the book, pays the second highest bid. That is, if player $i$ bids $b_i$, then player $i$ gains $v_i - b^*$ where $b^*$ is the second highest bid, $(v_i - b^*)/2$ if $i$ ties with one other for highest bid, and so on.

**Variation 3.** Similar, but highest bidder, while getting the book, pays the third highest bid. That is, if player $i$ bids $b_i$, then player $i$ gains $v_i - b^*$ where $b^*$ is the third highest bid, $(v_i - b^*)/2$ if $i$ ties with one other for highest bid, and so on.
NY City game and Schelling’s focal points

<table>
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<th>A</th>
<th>B</th>
<th>C</th>
<th>...</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1,1)</td>
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<td>(0,0)</td>
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<td>(0,0)</td>
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<tr>
<td>B</td>
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<td>(1,1)</td>
<td>(0,0)</td>
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<td>(1,1)</td>
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Attack-Defense game:

Variation 1. \( v_1 > v_2 > 0 \)

\[
\begin{array}{c|c|c}
\text{D1} & \text{D2} \\
\hline
\text{A1} & (0, 0) & (+v_1, -v_1) \\
\text{A2} & (+v_2, -v_2) & (0, 0) \\
\end{array}
\]

\( S_1 = \{A1, A2\}, S_2 = \{D1, D2\} \). Are there any Nash equilibria? If so, what are they?

Variation 2. \( v_1 > v_2 > 0 \)

\[
\begin{array}{c|c|c}
\text{D1} & \text{D2} \\
\hline
\text{A1} & (0, 0) & (+v_1, -v_1) \\
\text{A2} & (+v_2, -v_2) & (0, 0) \\
\end{array}
\]

\( S_1 = \text{probability distributions} \) on \( \{A1, A2\} \), \( S_2 = \text{probability distributions} \) on \( \{D1, D2\} \). Are there any Nash equilibria? If so, what are they? Payoffs are expected utilities.