A. (Autarky) Firm A in the US is the only seller of wine there. Let \( Q^A_{US} \) be Firm A’s sales in the US where demand is given by \( 6 – (1/8)Q^A_{US} \). Firm A’s costs are \( 1 \) per unit.

\[
\pi^A(Q^A_{US}) = (6 – (1/8)Q^A_{US}) \cdot Q^A_{US} - Q^A_{US}
\]

What value of \( Q^A_{US} \) maximizes profit? What is the sum of consumer surplus and firm profit?

B. (Free Trade) Now trade is opened between the US and France. Firm A is still the only producer in the US, while firm B is the only producer in France. Both can sell in either country: \( Q^A_{US} \) is A’s sales in the US; \( Q^A_{Fr} \) is A’s sales in France; \( Q^B_{US} \) is B’s sales in the US; \( Q^B_{Fr} \) is B’s sales in France. If a firm sells in the other country, their costs include not only their production costs but also a transportation cost of \( 1 \) per unit. Demand in the US is \( 6 – (1/8)(Q^A_{US} + Q^B_{US}) \) while demand in France is \( 6 – (1/8)(Q^A_{Fr} + Q^B_{Fr}) \). There is an important separability here; e.g.,

\[
\pi^A(Q^A_{US}, Q^A_{Fr}, Q^B_{US}, Q^B_{Fr}) =
\]

\[
P_{US}(Q^A_{US} + Q^B_{US}) \cdot Q^A_{US} + P_{Fr}(Q^A_{Fr} + Q^B_{Fr}) \cdot Q^A_{Fr} - (Q^A_{US} + Q^A_{Fr}) - Q^A_{Fr}
\]

\[
= [6 – (1/8)(Q^A_{US} + Q^B_{US})]Q^A_{US} + [6 – (1/8)(Q^A_{Fr} + Q^B_{Fr})]Q^A_{Fr} - Q^A_{US} - 2Q^A_{Fr}
\]

\[
= \{[6 – (1/8)(Q^A_{US} + Q^B_{US})]Q^A_{US} - Q^A_{US} \} + \{[6 – (1/8)(Q^A_{Fr} + Q^B_{Fr})]Q^A_{Fr} - 2Q^A_{Fr} \}
\]

\[
= \pi^A_1(Q^A_{US}, Q^B_{US}) + \pi^A_2(Q^A_{Fr}, Q^B_{Fr})
\]

Assume A and B act as Cournot duopolists in each of the US and French markets. What are the Nash equilibrium values of \( Q^A_{US}, Q^A_{Fr}, Q^B_{US}, \) and \( Q^B_{Fr} \)? What is the value of the sum (US consumer surplus + Firm A’s profit)?