Economics 611 Game Theory  Spring 2007  First Exam

All Syracuse University policies and procedures concerning academic honesty apply to this course:

"Syracuse University students shall exhibit honesty in all academic endeavors. Cheating in any form is not tolerated, nor is assisting another person to cheat. The submission of any work by a student is taken as a guarantee that the thoughts and expressions in it are the student's own except when properly credited to another. Violations of this principle include: giving or receiving aid in an exam or where otherwise prohibited, fraud, plagiarism, the falsification or forgery of any record, or any other deceptive act in connection with academic work. 

Plagiarism is the representation of another's words, ideas, programs, formulae, opinions, or other products of work as one's own either overtly or by failing to attribute them to their true source."

(Section 1.0, University Rules and Regulations)

WARNING!!!
While homework problems may have be done cooperatively, exams are individual work. Do not communicate about this exam with anyone except the instructor [x3-2345 or e-mail to jskelly@maxwell.syr.edu]. Violation of this rule will result in a grade of 0 for the exam. Any notices will be sent to you by e-mail; check occasionally.

EXPLAIN your answers carefully.

DUE: 9:30 am, Thursday, February 22, in class.
The game of red hats takes place with ten prisoners. That at least one hat is red is known up to level 1.

A. What happens if exactly two hats are red?
B. What happens if exactly three hats are red?
C. What happens if all ten hats are red?

A law is passed requiring a monopolistic soft-drink manufacturer to separate the production department and the marketing department. The marketing department chooses the price $P \geq 0$ to charge for a bottle of the firm’s soft drink and the production department chooses the level of output $Q \geq 0$. The two departments are forbidden to discuss their decisions with each other and, therefore, move simultaneously. Managers in both departments own shares in the firm and want to maximize its profits

$$\pi = P \cdot S - \frac{1}{2}Q^2$$

where $S$ denotes the firm’s sales. Sales can not exceed the firm’s output, nor can they exceed the market demand. Unsold output is thrown away. This means $S = \min\{Q, D(P)\}$ where market demand is

$$D(P) = 6 - P \text{ if } P \leq 6 \text{ and } D(P) = 0 \text{ if } P > 6$$

Find ALL Nash equilibria.

For the centipede game, determine ALL Nash equilibria. How many are there?
4. Player #1, the government, wishes to influence the choice of player #2. Player #2 will choose an action $a_2 \in A_2 = \{0, 1\}$ and will receive a transfer $t \in T = \{0, 1\}$ from the government, which observes $a_2$. Player 2 wishes to maximize the value of his transfer net of costs. The cost is 0 if $a_2 = 0$ and cost is $1/2$ if $a_2 = 1$. Player #1's objective is to minimize the sum $2(a_2 - 1)^2 + t$.

Before Player #2 chooses his action, the government announces a transfer rule $t(a_2)$ mapping $A_2$ to $T$. After player #2 chooses his action, the government chooses a transfer.

A. Draw the extensive form (game tree) for the case where the government’s announcement is not binding on the transfer chosen at the last stage. Then find all SPNE.

B. Draw the extensive form for the case where the government is constrained to implement the transfer rule it announced. Then find all SPNE.

5. At time 0, an incumbent firm (firm I) is already in the widget market, and a potential entrant firm (firm E) is considering entry. In order to enter, firm E must incur a cost of $K > 0$. Firm E’s only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game in the figure below is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight. Once firm E plays “out”, it is out of the market forever; firm E earns 0 in any period during which it is out of the market, and firm I earns $x$. The discount factor for both firms is $\delta$; i.e., a dollar gained in period 1 is valued at $\$1$; a dollar earned in period 2 is valued at $\delta \$1$; and a dollar earned in period 3 is valued at $\delta^2 \$1$. Similarly for costs.

Assume:
1. $x > z > y$;
2. $y + \delta x > (1 + \delta) z$;
3. $1 + \delta > K$;
4. $0 < \delta < 1$.

A. What are the SPNE of this game?
B. Is there a NE that is not a SPNE?