Economics 611  Game Theoretic Microeconomics
Spring 2010  First Exam

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**WARNING!!!**

While homework problems may have been done cooperatively, **exams are individual work.** Do not communicate about this exam with anyone except the instructor [x3-2345 or e-mail jskelly@maxwell.syr.edu]. **Violation of this rule will result in a grade of 0 for the exam.** Any notices will be sent to you by e-mail; check occasionally.

**EXPLAIN** your answers carefully.

**DUE: 9:30 am, Thursday, February 18, in class.**
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**EXPLAIN** your answers carefully. **DUE:** 9:30 am, Thursday, February 18, in class. The four problems are each worth 25 points.

1. a. (Autarky) (5 pts) Firm A in the US is the only seller of wine there. Let \( Q_{US}^A \) be Firm A’s sales in the US where demand is given by \( 11 - (1/4)Q_{US}^A \). Firm A’s costs are 1 per unit.

\[
\pi^A(Q_{US}^A) = (11 - (1/4)Q_{US}^A) \cdot Q_{US}^A - Q_{US}^A
\]

What value of \( Q_{US}^A \) maximizes profit? What is the sum of consumer surplus and firm profit?

b. (Free Trade) (20 pts) Now trade is opened between the US and France. Firm A is still the only producer in the US, while firm B is the only producer in France. Both can sell in either country: \( Q_{US}^A \) is A’s sales in the US; \( Q_{Fr}^A \) is A’s sales in France; \( Q_{US}^B \) is B’s sales in the US; \( Q_{Fr}^B \) is B’s sales in France. If a firm sells in the other country, their costs include not only their production costs but also a transportation cost of 1 per unit.

Demand in the US is \( 11 - (1/4)(Q_{US}^A + Q_{US}^B) \) while demand in France is \( 11 - (1/4)(Q_{Fr}^A + Q_{Fr}^B) \). There is an important separability here; e.g.,

\[
\pi^A(Q_{US}^A, Q_{Fr}^A, Q_{US}^B, Q_{Fr}^B) =
\]

\[
P_{US}(Q_{US}^A + Q_{US}^B) \cdot Q_{US}^A + P_{Fr}(Q_{Fr}^A + Q_{Fr}^B) \cdot Q_{Fr}^A - (Q_{US}^A + Q_{Fr}^A) - Q_{Fr}^A
\]

\[
= [11 - (1/4)(Q_{US}^A + Q_{US}^B)] \cdot Q_{US}^A + [11 - (1/4)(Q_{Fr}^A + Q_{Fr}^B)] \cdot Q_{Fr}^A - Q_{US}^A - 2Q_{Fr}^A
\]

\[
= \{[11 - (1/4)(Q_{US}^A + Q_{US}^B)] \cdot Q_{US}^A - Q_{US}^A \} + \{[11 - (1/4)(Q_{Fr}^A + Q_{Fr}^B)] \cdot Q_{Fr}^A - 2Q_{Fr}^A \}
\]

Assume A and B act as Cournot duopolists in each of the US and French markets. What are the Nash equilibrium values of \( Q_{US}^A, Q_{Fr}^A, Q_{US}^B, \) and \( Q_{Fr}^B \)? What is the value of the sum (US consumer surplus + Firm A’s profit)?
2. (25 pts) A law is passed requiring a monopolistic soft-drink manufacturer to separate the production department and the marketing department. The marketing department chooses the price $P \geq 0$ to charge for a bottle of the firm’s soft drink and the production department chooses the level of output $Q \geq 0$. The two departments are forbidden to discuss their decisions with each other and, therefore, move simultaneously. Managers in both departments own shares in the firm and want to maximize its profits

$$\pi = P \cdot S - \left[\frac{1}{2}Q^2 + Q\right]$$

where $S$ denotes the firm’s sales. Sales can not exceed the firm’s output, nor can they exceed the market demand. Unsold output is thrown away. This means $S = \min\{Q, D(P)\}$ where market demand is

$$D(P) = 6 - P \text{ if } P \leq 6 \text{ and } D(P) = 0 \text{ if } P > 6.$$  

A. Determine the best response correspondence for each department.

B. Find ALL Nash equilibria.

3. (25 pts) Consider the Mary-Tom auction game (the game tree is on the next page). Find all subgame perfect Nash equilibria. Is there a Nash equilibrium that is not subgame perfect?

4. (25 pts) There are two players and a status quo point $s_0 \in \mathbb{R}$. Player #1 offers a point $s_i \in \mathbb{R}$. Player #2 can then accept $s_i$ or reject it; in the latter case the outcome is $s_0$. If #2 accepts $s_i$, the outcome is $s_i$. Player #2's preferences are represented by $-(s - b_2)^2$ where $b_2$ is #2's bliss point, while player #1's preferences are represented by $-(s - b_1)^2$, where $b_1$ is #1's bliss point. Find the subgame perfect Nash equilibrium where all ties are broken in favor of the leftmost alternative in the tie.

Treat the cases
1. $s_0 < b_1 < b_2$
2. $b_1 < s_0 < b_2$
3. $b_1 < b_2 < s_0$
4. $s_0 < b_2 < b_1$