All Syracuse University policies and procedures concerning academic honesty apply to this course:

"Syracuse University students shall exhibit honesty in all academic endeavors. Cheating in any form is not tolerated, nor is assisting another person to cheat. The submission of any work by a student is taken as a guarantee that the thoughts and expressions in it are the student's own except when properly credited to another. Violations of this principle include: giving or receiving aid in an exam or where otherwise prohibited, fraud, plagiarism, the falsification or forgery of any record, or any other deceptive act in connection with academic work. Plagiarism is the representation of another's words, ideas, programs, formulae, opinions, or other products of work as one's own either overtly or by failing to attribute them to their true source." (Section 1.0, University Rules and Regulations)

WARNING!!!

While homework problems may have been done cooperatively, exams are individual work. Do not communicate about this exam with anyone except the instructor [x3-2345 or e-mail to jskelly@maxwell.syr.edu]. Violation of this rule will result in a grade of 0 for the exam. Any notices will be sent to you by e-mail; check occasionally.

EXPLAIN your answers carefully.

*Keep a Xerox copy of your answers to the take-home portion of your exam*

Take-home portion DUE: Noon, Friday, April 7.
EXPLAIN your answers carefully.

1. (Public goods mechanism design) [20 points]

Individuals have quasilinear preferences given by $u_i(x, \theta_i) = k\theta_i + (m_i + t_i)$.

Where $\theta = (\theta_1, \theta_2, \ldots, \theta_I)$, suppose a social choice function $f$ with

$$f(\theta) = (k(\theta), t_1(\theta), t_2(\theta), \ldots, t_I(\theta))$$

that satisfies

(A) $k(\theta) = \begin{cases} 
1 & \text{if } \sum \theta_i \geq c \\
0 & \text{otherwise}
\end{cases}$

(B) $\sum t_i(\theta) = -ck(\theta)$.

Revelation mechanism with income tax:

Choose $k(\tilde{\theta}) = 1$ if $\sum \tilde{\theta}_i \geq c$; 0 otherwise.

$$t_i(\tilde{\theta}) = -\left(\frac{c m_i}{\sum m_i}\right)k(\tilde{\theta})$$ [You pay in proportion to your income. The $m_i$ values are common knowledge.]

Assume $\Theta_i = \{\theta_i\}$ for $i$ not equal to 1; $\Theta_i = \mathbb{R}_+$.

A. Show, by an example, that for some values of $m_1, \ldots, m_I, c, \theta_1, \ldots, \theta_I$, individual 1 has an incentive to understate his preference for the public good, by choosing $\tilde{\theta}_i < \theta_i$. Extra points for an example where this causes $\sum \tilde{\theta}_i < c$ even though $\sum \theta_i \geq c$.

B. Show, by a different example, that for some values of $m_1, \ldots, m_I, c, \theta_1, \ldots, \theta_I$, individual 1 has an incentive to overstate his preference for the public good, by choosing $\tilde{\theta}_i > \theta_i$. Extra points for an example where this causes $\sum \tilde{\theta}_i \geq c$ even though $\sum \theta_i < c$. 
2. (Public goods mechanism design) [20 points]

With the same utility functions as in question 1, now consider the revelation mechanism

Choose \( k(\theta) = 1 \) if \( \sum \tilde{\theta}_i \geq c \); 0 otherwise.

\[ t_i(\theta) = \sum_{j \neq i} \tilde{\theta}_{ij} - c. \]

Assume \( \Theta_i = \mathbb{R}_+ \) for all \( i \). In class I said that for this rule, no one individual has an incentive to choose a \( \tilde{\theta}_i \) different from \( \theta_i \). Show by an example that when everyone else is telling the truth, two individuals both choosing a \( \tilde{\theta}_i \) different from their \( \theta_i \) can improve their outcome over what they get if they tell the truth. Give specific numerical values for \( \theta_1, \theta_2, \ldots, \theta_i \) and \( c \). Extra points for an example where this causes \( \sum \tilde{\theta}_i < c \) even though \( \sum \theta_i \geq c \), or where this causes \( \sum \tilde{\theta}_i \geq c \) even though \( \sum \theta_i < c \).

3. (Gibbard-Satterthwaite) [20 points]

There are, say, 3 alternatives and 6 individuals, each with strong orderings over the alternatives. For each of the following two social choice rules present a preference profile at least someone has an incentive to misrepresent their preference ordering.

**Rule I.** If one alternative is at the top of the preference ordering for more individuals than is true for any other alternatives, that alternative is chosen. If there is a tie between two or more alternatives for the largest number of top positions, the alphabetically earlier of those is selected by Rule I.

**Rule II.** Look at the top two alternatives for individual #1. Have a majority vote by the remaining 5 individuals, 2, 3, ..., 6 and the rule selects the majority winner.

For each rule, present a preference profile at least someone has an incentive to misrepresent their preference ordering.
Take-home portion DUE: Noon, Thursday, April 6, in class.

4. (Weierstrass) [20 points]

A fishery earns a profit of \( \pi(x) \) in a year from catching and selling \( x \) fish in that year. The firm owns a pool which currently has \( y_1 \) fish in it. (Note: \( y_1 \) is a fixed parameter.) If \( x \in [0, y_1] \) fish are caught this period, the remaining \( y_1 - x \) fish will grow to \( f(y_1 - x) \) by the beginning of the next period, where \( f: \mathbb{R}_+ \to \mathbb{R}_+ \) is the growth function for the fish population. The fishery wishes to set the volume of its catch in the next three periods to maximize the sum of its (undiscounted) profits over this time. That is, it wishes to solve the problem of maximizing with respect to \( x_1, x_2, \) and \( x_3 \), the sum \( \pi(x_1) + \pi(x_2) + \pi(x_3) \) subject to \( x_1, x_2, x_3 \geq 0, \)

\[
  x_1 \leq y_1; \quad x_2 \leq y_2 = f(y_1 - x_1); \quad x_3 \leq y_3 = f(y_2 - x_2).
\]

The terms \( y_2 \) and \( y_3 \) are just for exposition; they are not additional variables or parameters; we could have written the constraints as:

\[
  x_1 \leq y_1; \quad x_2 \leq f(y_1 - x_1); \quad x_3 \leq f(f(y_1 - x_1) - x_2).
\]

Assume that the functions \( \pi \) and \( f \) are continuous on \( \mathbb{R}_+ \), and show that there does exist a global maximum for the firm’s problem. Do NOT assume that \( \pi \) and \( f \) are increasing functions of their arguments.
5. (Moral hazard with endogenous probability of detection) [20 points]

Consider an extension of our basic principal-agent model of employment. This time, the firm first selects a high or low level of surveillance. The low level of surveillance, which is costless, results in a probability of workers getting caught shirking of \( \pi(E) = 1 - E \). The high level of surveillance, which entails fixed costs \( C \), results in a probability of workers getting caught shirking of \( \pi(E) = 1 - E^2 \). Then the firm announces a \((w, L)\) contract. The worker then chooses whether or not to accept the contract and, if it accepts, chooses a level of effort, \( E \). The firm has revenue function: \( V(L, E) = V(L \cdot E) = (L \cdot E)^\theta \) and labor costs of \( W \cdot L \). Employees have utility function \( U = W \cdot (1 - E) \) and exogenous reservation utility: \( W \).

Assuming that employees can observe the surveillance level before they select their effort level, what are the subgame perfect equilibria?

*EXPLAIN* your answers carefully.