Probability Theory

Experiment

Outcome

Sample Space, S: Set of all possible outcomes

Event: Subset E of the Sample Space, S

Probability Function: A function P from events to the real numbers such that:

1. \( P(E) \geq 0; \)
2. \( P(S) = 1; \)
3. \( A \cap B = \emptyset \) implies \( P(A \cup B) = P(A) + P(B). \)

A\(^c\): Complement of A: \( A \cup A^c = S \) and \( A \cap A^c = \emptyset \)

So \( P(A) + P(A^c) = P(A \cup A^c) = P(S) = 1 \)

\( P(A^c) = 1 - P(A) \)

\[ P(A) + P(A^c) = 1 \]

\( P(A) \leq 1 \)

\( P(\emptyset) = 0 \)

If \( E_1 \cap E_2 = \emptyset \) and \( E_2 \cap E_3 = \emptyset \) and \( E_1 \cap E_3 = \emptyset \), then

\[ P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) \]

If \( E_1, E_2, ..., E_i \) are pairwise disjoint, then

\[ P(E_1 \cup E_2 \cup ... \cup E_i) = P(E_1) + P(E_2) + ... + P(E_i) \]

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
Elementary Events

Equiprobable elementary events: \( P(A) = \frac{\#A}{\#S} \)

Fundamental Counting Rule:

1. Suppose a process can be carried out in two stages; the first stage can be carried out in \( m \) ways while the second stage can be carried out in \( n \) ways. Then the whole process can be carried out in \( m \times n \) ways.

2. Suppose a process can be carried out in three stages; the first stage can be carried out in \( m \) ways while the second stage can be carried out in \( n \) ways and the third stage can be carried out in \( r \) ways. Then the whole process can be carried out in \( m \times n \times r \) ways.

Examples:

1. Number of outcomes of 10 successive coin flips
2. Number of five letter "words"
3. Number of possible: six entry alpha-numeric license plates
4. With ordinary poker deck, the number of five card dealings \[311,875,200\]
5. The number of permutations of \( n \) things taken \( r \) at a time
6. With ordinary poker deck,
   i. The number of two card hands;
   ii. The number of three card hands;
   iii. The number of five card hands. \[2,598,960\]
7. The number of combinations of \( n \) things taken \( r \) at a time.
Poker deck

52 cards, distinguished by color, suit, and denomination

26 Red: Suits are hearts (13) and diamonds (13);
26 Black: Suits are spades (13) and clubs (13).

13 denominations within each suit: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q), King (K), Ace (A)

Poker hands

Royal flush: 10-J-Q-K-A in one suit

Straight flush: sequential denominations in one suit

Four of a kind

Full house: Three of one denomination, two of another

Flush: one suit

Straight: sequential denominations

Three of a kind

Two pair

One pair

Not even one pair or better