Ordinary Least Squares (OLS): Simple Regression

\[ y_t = \hat{y}_t + \hat{\epsilon}_t = a + bx_t + \hat{\epsilon}_t \]

\[ \hat{\epsilon}_t = y_t - \hat{y}_t \]

\[ \min_{a, b} V(a, b) = \sum_{1}^{T} (\hat{\epsilon}_t)^2 = \sum_{1}^{T} [y_t - \hat{y}_t]^2 = \sum_{1}^{T} [y_t - (a + bx_t)]^2 \]

\[ \frac{\partial V}{\partial a} = \sum_{1}^{T} 2[y_t - (a + bx_t)][-1] \]

\[ \frac{\partial V}{\partial b} = \sum_{1}^{T} 2[y_t - (a + bx_t)][-x_t] \]

\[ \sum_{1}^{T} [y_t - (a + bx_t)] = 0 \quad \text{i.e., } \sum \hat{\epsilon}_t = 0 \]

\[ \sum_{1}^{T} [y_t - (a + bx_t)][-x_t] = 0 \quad \text{i.e., } \sum x_t \hat{\epsilon}_t = 0 \]

\[ \sum_{1}^{T} y_t \quad \sum_{1}^{T} a \quad \sum_{1}^{T} bx_t = 0 \]

\[ \sum_{1}^{T} x_t y_t \quad \sum_{1}^{T} a x_t \quad \sum_{1}^{T} bx_t^2 = 0 \]
Normal equations:

\[
Ta + b \Sigma x_t = \Sigma y_t \\
\Sigma x_t^T a + b \Sigma [x_t]^2 = \Sigma x_t y_t
\]

**OLS ESTIMATORS:**

\[
b = \frac{T \cdot (\Sigma x_t^T y_t) - (\Sigma x_t)^T (\Sigma y_t)}{T \cdot (\Sigma x_t^2) - (\Sigma x_t)^2} = \frac{\Sigma (x_t - \bar{x})(y_t - \bar{y})}{\Sigma (x_t - \bar{x})^2}
\]

\[
a = \bar{y} - b \bar{x}
\]

Suppose

\[
y_t = \alpha + \beta x_t + e_t
\]

where each \(e_t\) is normally distributed with mean 0 and variance \(\sigma^2\). Then \(b\) is normally distributed with mean \(\beta\) and variance

\[
\sigma^2 \frac{T}{\Sigma \left( X_t - \bar{X} \right)^2}
\]
Hence

\[
\frac{b - \beta}{\sigma / \sqrt{\Sigma (X_t - \bar{X})^2}}
\]

has a standard normal distribution.

If we estimate \( \sigma \) by

\[
s = \sqrt{\frac{T}{T - 2} \frac{\Sigma e_t^2}{T - 1}}
\]

then

\[
\frac{b - \beta}{s / \sqrt{\Sigma (X_t - \bar{X})^2}}
\]

has a Student's t-distribution with \( T - 2 \) degrees of freedom.